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Predicate-term negation and the indeterminacy of the future

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Abstract This essay introduces a formal structure to model the indeterminacy of the future in Einstein-Minkowski space-time. We consider a first-order language, supplemented with an operator for predicate-term negation, and defend the claim that such an operation provides an appropriate model for the indeterminacy of future contingents. In the final section, it is proved that given a language otherwise adequate to represent a physical theory, at least some of the predicates of that language are indeterminate when the future is not causally determined by the present and the past.

Keywords Future Contingents · Spacetime

1 Introduction

In this essay, I prove that there is a meaningful "match" or "consilience" between two independently plausible ways of representing the openness of the future in relativistic space-times. One, semantic, model for the indeterminacy of the future provides a plausible explication of the classical problem of future contingents, without the need to postulate either additional space-time structure, not required by relativistic physics, or non-standard first order logic.

Consider Aristotle's classical worry about disjunctions such as "Either there will or there will not be a sea-battle tomorrow." This, and similar puzzles, raise the specter of logical fatalism. However, such concerns also raise serious worries about the causal efficacy of human decisions, among other events. If the sea-battle must, or must not, as occur as a matter of logic, isn't the commander sitting in his tent brooding over whether to offer battle or run away simply wasting his or her time and effort? How can their deliberation and ultimate decision be part of bringing it about that the battle occurs? However,I also prove that the relation of causal connection on Einstein-Minkowski space-time provides us with an independent characterization of what it means for the past, or the past and the present, to fail to "fix" the physical state of the future. In §4, this conception of indeterminacy, *relational indeterminacy*, is defined, and it is shown that for a certain class of plausible cases such relational indeterminacy implies semantic indeterminacy.

Most of the contemporary effort at explicating the openness of the future has focused on the logical face of this problem and in this paper I offer a new solution to this problem; one not previously discussed and having substantial advantages over alternative solutions. Even if the reader is not persuaded of those virtues, it is not that often that a completely new proposal to solve a classical philosophical problem

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appears. I propose that the future differs from the past in that future objects or events manifest attribute indefiniteness; there are some properties that future events or objects *could* possess that they neither have nor lack.[1]

In the following two sections I make this conception of an entity neither having nor lacking a particular attribute at least somewhat rigorous and illustrate how we can use it to solve the classical logical problem of future contingents. I also argue that as a solution it is at least as plausible as the stanrdard alternatives such as appeals to three-valued logics, truth value gaps or modally branching time structures. The most important advantage to this conception of the open future doesn't appear until section 4 where I prove that this conception of the openness of the future provides a natural interpretation of the variety of indeterminacy characteristic of the failure of the past to causally determine the future. Even more strikingly this semantic indeterminacy *follows* from special relativity given just a few plausible additional assumptions. And, there simply is no other conception of the openness of the future on offer that is straightforwardly compatible with relativistic theories of space-time.

2 Predicate Term Negation as Model of Indeterminacy

One of the most basic intuitions about the indeterminate future is that there is something"fuzzy" about entities to the future. Even if they exist, it is far from clear what properties they possess or what events they are involved in; they suffer from what Adolf Grünbaum has called "attribute indefiniteness"(Grünbaum 1963). How might we model this "fuzziness?" Consider, my weight in 6 months. If my future weight is indeterminate, it seems wrong to say that I weigh 215 pounds in six months, but it seems equally wrong to claim that I don't weigh that, especially where that seems imply that there is some other weight that I do have. In standard first-order logic we interpret a sentence involving the predicate "...does not weigh 215 lbs." as the truth-functional negation of one involving "...weighs 215 pounds." However, we need not bind ourselves to that interpretation. There is at least some intuitive appeal to distinguishing between having the property of not weighing 215 and not having the property of weighing 215. We can represent that distinction by introducing a language which contains operators that convert predicates into new predicates in addition to the usual sentential operators of first-order logic.

Let us begin with a standard first-order logical language, $\mathscr{L}_{\mathrm{T}}$, consisting of a list of singular terms $\mathrm{t}_1 \cdots \mathrm{t}_{\mathrm{n}}$ and n-ary predicates, $\mathrm{P}^{\mathrm{n}}_1 \cdots \mathrm{P}^{\mathrm{n}}_{\mathrm{i}}$ the universal quantifier, $\forall$, negation,$\sim$, conjunction, &. The formation rules and valuation for $\mathscr{L}_{\mathrm{T}}$ are completely standard. However, as I noted above, it has long been recognized that the sentential operators of such languages can be supplemented with a further set of operators, *predicate operators*(cf. Prior and Fine 1977). For our purposes, we only need one such operator, to generate the language $\mathscr{L}_{\mathrm{TN}}$, a predicate negation operator. This will allow us to clarify an important sense in which there is a distinction between *denying* that an entity possesses a particular property and *asserting* that the same entity has the contrary property. Finally, and purely as a matter of notational convenience, we will help ourselves to a future tense operator, $\boldsymbol{F}$, as shorthand to "In the future, ...".

The predicate negation operator obeys the following straightforward formation rule:

$$\mathbf{N}\mathrm{P}^n_i \text{is an n-ary predicate of } \mathscr{L}_{\mathrm{TN}} \text{ if and only if } \mathrm{P}^n_i \text{ is.} \tag{2.1}$$

As usual the models, $\mathscr{M}$, of $\mathscr{L}_{\mathrm{TN}}$consist of a domain, $\mathbb{D}$, a valuation function, $\mathbf{v}$ and an interpretation. The interpretation of the sentences of $\mathscr{L}_{\mathrm{TN}}$is just that of ordinary first-order logic, but the valuation needs to be tweaked to account for our predicate-term negation operator:

Definition 1 $\mathbf{v}$ *is a valuation function that assigns to each t_i in $\mathscr{L}_{TN}$ an element $\delta_i \in \mathbb{D}$ and to each P^n_i an element $\mathscr{P}(\mathbb{D}^n)$, with the additional constraint that $v(\boldsymbol{N}P^n_i) \bigcap v(P^n_i) = \emptyset$.*

Thus, the operator '**N**' operates as, in the language of neo-Aristotelian term logic, a variety of *predicate-term* negation.[2] Now consider the difference between '∼Pt' and '**N**Pt' for some one place predicate. These are logically equivalent if and only if '**NP**' designates the logical contrary to a predicate. Let us say that a predicate such that this is the case, according to a particular model, is *determinate in* $\mathscr{M}$. That is:

Definition 2

$$P^n_i \textit{ is } \boldsymbol{determinate\ in}\ \mathscr{M} \textit{ if and only if } \boldsymbol{v}[\mathscr{M}, P^n_i] \cup \boldsymbol{v}[\mathscr{M}, \boldsymbol{N}P^n_i] = \mathbb{D}^n$$

However, nothing in the concept of predicate negation seems to require that all of the predicates of our language be determinate in all of our models in this way. In addition, there are good reasons to weaken this restriction. First, because most predicates, *e.g.* colors, do possess a range of non-logical contraries. Second, given a collection of predicates representing what we take to be intrinsic or "projectible" properties, there does seem to be a "natural" contrary to each such predicate read as "definitely does not possess that property or relation."

When we weaken the requirement that all predicates be determinate in this sense, we gain a useful framework for making sense of the indeterminacy of the future. First, some terminology will be useful. We will say that a given predicate is *indeterminate* in a model just in case it is not determinate in the sense of Definition 2 although we will often leave the restriction to a model implicit. We will also sometimes write of indeterminate properties or relations, when, literally speaking, the corresponding predicate is indeterminate. Finally, say that particular entity *manifests* the indeterminacy of an attribute, when that entity falls into the extension gap of the predicate for that attribute. For an entity that manifests the indeterminacy of redness, for example, both the assertion that the predicate, "... is red," holds of a particular entity and that its predicate-denial, "...is not-red," does are false, and the sentential-negation of both the assertion and the predicate-denial are true. Thus, the basic intuition about "attribute-indefiniteness" seems to be captured precisely by the idea that an object neither definitely possesses nor definitely lacks a property (stands or fails to stand in a particular relation). More formally, consider a monadic predicate P^n_i which is indeterminate, then there is some $\delta \in \mathbb{D}$ such that both $P_i(\mathrm{t})$ and $\mathbf{N}\mathrm{P}_i(\mathrm{t})$ are false. In the next section, I will argue that this is at least as plausible a theory of future contingents as those previously available.

3 Future contingents as contingently false

In the previous section, I presented a new conception of indeterminacy using predicate-term negation. As a model of indeterminacy, I hope the intuitive plausibility is obvious. If my weight in six months is indeterminate, then it should be false to claim that my weight in six months has any particular value. Unfortunately, that by itself doesn't solve our problem. If it is false that my weight in six months is 215 pounds, then it is *true* that my weight in six months is *not* 215 pounds. This is where the predicate-term negation comes into play; it allows us to distinguish between denying the truth of a sentence and asserting the contrary of that sentence. This allows that it might be that:

$$\text{It is not the case that my weight in six months is 215 pounds.} \tag{3.1}$$

while also allowing that it is false to claim that:

$$\text{My weight in six months has some value other than 215 pounds.} \tag{3.2}$$

This should be sufficient to persuade one that predicate indeterminacy is a viable competitor to the standard accounts of indeterminacy already appearing in the literature, including three, or more, valued logics(See, e.g. Prior 1953); truth-value gaps(See, e.g. Thomason 1970); and branching time or space-times theories(See, e.g., Belnap 1992; McCall 1976, 1994). Unfortunately, it seems unlikely that one would be able to identify a knock-down argument in favor of any of the, now four, available alternatives. Too much of the decision about which model to embrace will depend on judgements of "fit" with one's other theoretical commitments. The closest argument on offer is that in section 4 that demonstrates the connection between predicate indeterminacy and an apparently non-optional presupposition of any theory of space or time: Einstein's theory of relativity. However, before we consider that argument, there are three other considerations in favor of predicate indeterminacy. First, it provides a plausible account of the disjunction or "sea-battle" problem of future contingents and does so with less problematic presuppositions than the previously available alternatives. Second, that it does so against the backdrop of an at least potentially eternalist ontology. Third, that it provides an account not merely of indeterminacy, but also of what it means for an event to occur and become determinate.

Consider, once again, the problem posed by (**??**), rewritten in slightly different form below:

$$\text{Either a sea-battle will occur tomorrow, or a sea-battle will not occur tomorrow.} \tag{3.3}$$

As we have seen before there is a tension between the apparently necessary truth of (3.3) and the, at best, contingent truth of the disjuncts.

Any non-fatalist account of (3.3) needs to offer an alternative to the standard propositional logic interpretation either of the disjuncts or of disjunction. A predicate indeterminacy account begins by diagnosing a previously unrecognized ambiguity in the negated disjunct. We could read it as a straightforward sentential negation of the first disjunct:

$$\text{It is not the case that a sea-battle will occur tomorrow.} \tag{3.4}$$

Or, in $\mathscr{L}_{\mathrm{TN}}$, modeled in a domain consisting of independently identified events and predicates that attribute certain types to those events:

$$\sim \boldsymbol{F}\mathrm{St} \tag{3.5}$$

Here, we read "S" as "… is a sea-battle."[3] However, now consider the alternative, which we might loosely read as follows:

$$\text{Something other than a sea-battle will occur tomorrow.} \tag{3.6}$$

In $\mathscr{L}_{\mathrm{TN}}$, this could be:

$$\boldsymbol{F}\mathbf{N}\mathrm{St}^{4} \tag{3.7}$$

However, this seems a quite plausible distinction. (3.5) is the *present* denial that a sea-battle occurs in the future. Since the sea-battle has not actually occurred, and might not, this seems to be a true statement. (3.7), on the other hand, *now* asserts, about the future, that it will not be occupied by a sea-battle. Since nothing now determines whether that will be the case, is seems plausible that such an assertion is presently false. Putting this all together we get a resolution of the classical puzzle. If we interpret the second disjunct as the *contradictory* of the first disjunct, (3.5), then (3.3) is a tautology and is true, in the indeterminate case, just because the second disjunct is true. However, if we interpret the second disjunct according to (3.7), then the two disjuncts are *contraries* not contradictories, (3.3) is not a tautology, and in the indeterminate case is simply false. Ultimately, the predicate indeterminacy account of future contingents takes advantage of the standard natural language ambiguity between contradictory statements and merely contrary ones.

This is at best a plausibility argument and adopting the predicate indeterminacy account certainly forces us to pay a price by introducing second-order considerations into what had seemed like a first-order logical problem. However, the other three alternatives come with their own costs. First, consider a Lukaciewiz-style three-valued logic. On such an account, both disjuncts, despite being contradictories, would be assigned the same, indeterminate, truth-value. One then chooses either the "weak" or "strong" interpretation of disjunction. On the weak interpretation, where the truth-value of the disjunction is the maximum of the disjuncts, (3.3) is also indeterminate.On the strong interpretation of disjunction, (3.3) is true, but one is forced to deny excluded middle. The truth of the disjunction does not imply the truth of one of the disjuncts.

Alternatively, we might introduce truth-value gaps. On this approach, as long as it is undetermined whether there will be a sea-battle or not, neither of the disjuncts of (3.3) have any truth-value at all. However, (3.3) itself is true because that is its truth value on every consistent assignment of truth-values to its disjuncts. Both of these solutions suffer from the problem that they require radical changes to propositional logic. These are changes that ramify throughout our logic and require us to reinterpret the meaning of and inference rules for disjunction even in situations without any indeterminacy. And, it is the unwillingness to pay this kind of a logical price which seems to have driven many philosophers to take logical fatalism seriously. This is not a problem that afflicts predicate indeterminacy. While we do need to make a decision about whether to interpret a given sentence as either a sentential denial or a predicate denial, once we made that decision the semantic interpretation of the formalized sentence and the inference patterns that it supports are precisely those of ordinary first-order logic.

The final alternative, sometimes presented by its defenders as a development of Thomason's truth-value gap approach above, is the branching space-time program(See, e.g., Belnap 1992; McCall 1976, 1994). In addition to the above concerns, the branching space-times program poses what seem to be insurmountable problems in the physical theory of space-time. A full account of the concerns raised by such branching space-time structures would require a whole other essay.[5] However, here is one concern regarding branching space-time. All versions of Belnap's branching space-time(s) theories begin with a set of events, OWP. Although some of Belnap's recent statements seem to indicate that branching is a fundamentally modal notion, it remains true that branching occurs *at* elements of OW, for O(ur) W(orld). But, then each event where branching occurs has a very strange future. The set of other events which are future timelike connected to that event consists of at least two disjoint, and in fact topologically diconnected, sub-regions; one such region for each branch "leaving" the event.

What one should say next depends on how one answers the question of what a spacetime *is*? One natural construal of a space-time and of Belnap's position is that space-time *per se* consists in the entire collection of causally interconnected events. But, then we simply have no idea how to connect such structure to the kind of things that we study in physics that we also call space-times. In particular, this space-time, the whole branching structure, is not Hausdorff. In fact, it's not even clear that it's a manifold.We also seem to fall afoul of various other theorems, for example regarding topology change. In more recent publications, Belnap and his collaborators have suggested that space-time ordinarily conceived is *not* the whole structure, but that each maximally consistent history within the structure constitutes a space-time in the physicists sense.(See, for example Placek and Belnap 2010, §§2.7, 3.1) But, this would mean that every branching event occupies more than one space-time. I, at least, am not even sure what it means to claim that one has two space-times containing the same, numerically identical, event. Until we have some kind of detail regarding how to connect this "pre-physical" picture of the collection of point events with existing physical theories of the structure of the manifold of point events, one is left wondering if branching space-time might not be merely an interesting logical-mathematical game.

Thus, all of the conceptions of indeterminacy on offer require us to pay a price, whether in complexity, logical messiness or coherence with the available physics. Although I do not claim that there is any absolute sense in which I have established the superiority of predicate indeterminacy, I have shown that predicate indeterminacy is *no more* problematic than the previously available alternatives.

In addition, it has additional advantages. First, the indeterminacy of predicates is independent of one's object-level ontology. In particular, it is independent of the dispute between presentist/growing block ontologies and eternalist ontologies. This seems exactly correct. It should be obvious that one can be a presentist in ontology while also being a fatalistic determinist; this model demonstrates that one can equally well be a non-fatalist eternalist. To see this, simply notice that the model structure in §2 makes use of single domain with unrestricted quantification over that domain. Second, it provides model for becoming determinate, as well as of indeterminacy. Consider some property, represented by the predicate P_i, that is indeterminate with respect to an object δ at time t_o. If the object later comes to possess the property, it is *literally* added to the extension of P_i. It provides, therefore, not merely an eternalist model of indeterminacy, but of becoming. Finally, the next section will demonstrate that it provides a model for indeterminacy in the Einstein-Minkowski space-time of special relativity.

4 Indeterminacy in Einstein-Minkowski Space-time

The final stage of the argument is to demonstrate that the *predicate indeterminacy* of the previous sections is a natural consequence of the *causal indeterminacy* that arises in relativistic space-time, namely the simplest such space-time of special relativity. This argument proceeds in two stages. First, we need reasonably precise specification of what it means for the future to fail to be determined by the past; what I will call *relational indeterminacy.* Second, we demonstrate that, given certain other plausible assumptions, relational indeterminacy in this sense is deeply associated with semantic or predicate indeterminacy. It is then trivial to demonstrate that the future of each space-time location in Einstein-Minkowski is relationally indeterminate and therefore manifests semantic indeterminacy.

What does it mean to claim that the future is not determined by the past? In order to remain at the greatest level of generality, consider space-time theories. Let's say that, abstractly, such a theory consists of a space-time and the assignment of possible values to every point of space-time. In the standard cases, assignments of scalar, vector or tensor fields to the space-time. A specification of all of the available types of values to each point of the space-time, I will call the state of the space-time, relative to a particular theory.[6] Now, let us consider topologically open regions of space-time, down to points. Obviously, given the state of space-time, the relevant values are also determined for each such region. In the absence of a dynamics, there is a meaningful sense in which all possible assignments of such values are equally allowable. But, of course, this is not what we want to know. We do not deal in God's eye views of space-time, except at the most abstract levels. We deal with regions of space-time.

What we really want to know is: given the (full or partial) specification of the state of a region of space-time, how does that, given a relevant theory, constrain the states of other regions of space-time? Obviously, to do this we need a dynamics that connects the states of different regions of space-time and connects the various "elements" of the state. What we seem to require is a specification of how likely a region R is to be in a particular state, r, given that some other region, S, is in a particular state, s. The obvious option is to locate some natural probability measure and use the conditional probability, where we say that the state of S determines the state of R when the conditional probability equals 1.[7]

The natural assignment seems to be to an equal probability to all dynamically consistent states of the entire space-time. Now consider an arbitrary region, an open set in the usual topology, R of the space-time. Some of the dynamically possible states of the space-time assign the same state to that region, some of them assign different states. Thus, the states of regions inherit the probability that they will be in a given state from the initial probability assignment to states of the space-time. That is, for all possible states r of R, we can derive the probability that R is in state r, $\text{prob}(r\equiv R)$. But, now consider another region, S. Via the usual definition of conditional probability, we can define for each state s of S, the probability that S is in s given that R is in r, $\text{prob}(s\equiv S|r\equiv R)$ for each of possible state r of R.

Finally, given that our goal is to investigate what regions of space-time are indeterminate relative to certain other regions in various space-times, the minimal constraints on the dynamics are the relations of causal connection appropriate to the space-time under consideration.

Definition 3 ***caus****(S,R) if and only if the state of S could causally influence the state of R.*

First, note that in these definitions regions can be replaced with points as the degenerate case of regions in the usual topology. Second, Definition 3 allows us to define a, not necessarily exhaustive, partition of the space-time, relative to any given region R, into the causal past, $\mathbb{C}$(R); the causal present, $\mathbb{P}$(R); and the causal future, $\mathbb{F}$(R), as follows:

Definition 4 $\forall q, R\ \exists p\{(q \in \mathbb{C}(R)) \Leftrightarrow (p \in R\ \&\boldsymbol{caus}qp\ \&\ \sim \boldsymbol{caus}pq)\}$

Definition 5 $\forall q, R\ \exists p\{(q \in \mathbb{P}(R)) \Leftrightarrow (p \in R\ \&\boldsymbol{caus}qp\ \&\ \boldsymbol{caus}pq)\}$

Definition 6 $\forall q, R\ \exists p\{(q \in \mathbb{F}(R)) \Leftrightarrow (p \in R\ \&\ \sim \boldsymbol{caus}qp\ \&\ \boldsymbol{caus}pq)\}$

It will also be useful to have a name for the entire region from which R is causally accessible, i.e. the union of $\mathbb{C}$(R) and $\mathbb{P}$(R): call it, $\mathbb{A}$(R). For clarity note that in relativistic space-times, assuming the standard connection between actual causal connectability and the geometry of the space-time, these regions consist of the future null-cone and its interior, the space-time location itself, and the past null-cone and its interior respectively.We are now ready to say what it is for the state of one region of space-time to determine the state of another region. As we should expect from the above:

Definition 7 *S **determines** R [**det**(S,R)] if and only if for each possible state s of S, there exists a state r of R, such that for some Q⊆S, Q⊆ $\mathbb{A}$(R) and for the state q of Q induced by s, prob(r≡R|q≡Q)=1.*

Now, we possess the necessary resources to define *relational indeterminacy*(RI). Given the above it must be that the future is open if it is not determined, in the sense of Def. 7, by the past. Which past? Given that the future is defined relative to the changing space-time location of entities along their world-lines, it must be relative to *their* past. Thus, define the following two concepts:

Definition 8 *A point q_o is **Relationally Indeterminate** (RI) relative to a point q_1 if and only if there is no R⊆ $\mathbb{A}(q_1)$ such that **det**(R,q_0)*

Definition 9 *A point q_o is **determinate** relative to a point q_1 if and only if there is an R⊆ $\mathbb{A}(q_1)$ such that **det**(R,q_0)*

Therefore, we can finally state that the future is open, in at least one significant sense, if and only if it is relationally indeterminate relative to our changing location. Or, alternatively, that an event happens at $\mathrm{q_o}$ relative to $\mathrm{q_1}$ only when the state of $\mathrm{q_o}$ is determinate relative to $\mathrm{q_1}$.

It is important to distinguish *relational indeterminacy* from two closely related concepts: *predictive indeterminacy* and *indeterminism.* Predictive indeterminacy is, as the name implies, a fundamentally epistemic notion. It is merely the claim that a particular epistemic agent, in a particular situation, lacks the resources to establish the state of a particular region of space-time. Karl Popper, for example, confuses predictive with relational indeterminacy in *The Open Universe*(Popper 1991). Note that while relational indeterminacy implies predictive indeterminacy, on the assumption that causally accessible information exhausts the information available to epistemic agents, the converse is not true.

Alternatively, *indeterminism* implies relational indeterminacy, but not *vice versa.* That is, consider a point, p such that the causal past *of p* fails to determine its state: there is some causal indeterminism in the universe. Then, from any region not containing p, p is relationally indeterminate. But, again, there are other sources of relational indeterminacy in the universe. For example, even in a deterministic universe, the future light-cone of every point in Einstein-Minkowski space-time is relationally indeterminate to that point. It is this relationship between indeterminism and relational indeterminacy that seems to have led Hans Reichenbach to insist on the relationship between indeterminism and temporal becoming(cf. Reichenbach 1991, 1925)

To show that *relational indeterminacy*, plus plausible assumptions, yields semantic indeterminacy, begin by considering a covering, $\mathcal{C}$, of the space-time extracted from the usual topology.[8] This covering now constitutes a domain for the model structure of section 2. Next, define what it is for the predicates of a language, $\mathscr{L}_\psi$, to provide an adequate representation of a space-time theory, $\mathbf{T}$, where$\{\psi_i\}$ is the set of possible states of the space-time according to $\mathbf{T}$, relative to $\mathcal{C}$. Let us say that:

Definition 10 *A set of predicates* $\mathbb{P} = \{P_1 \cdots P_n\}$ *is* adequate *to* $\boldsymbol{T}$ *and* $\mathcal{C}$ *if and only if for all* ψ *and all* $R \in \mathcal{C}$ $P_i R$ *for exactly one* $P_i \in \mathbb{P}$.

Intuitively, $\mathbb{P}$ provides a partition of the possible states of the regions specified in $\mathcal{C}$ such that we can use $\mathbb{P}$ to specify the state of the space-time to a precision limited by the precision with which we specify $\mathbb{P}$ and the size of the regions in $\mathcal{C}$. Call a language with such an adequate set of predicates, $\mathscr{L}_\psi$. Next,

Definition 11 *A* local interpretation *of* $\mathscr{L}_\psi$ *relative to a region, R, consists of a partial interpretation, with valuation* v_R*, such that for all* $s \in \mathcal{C}$,

1. $s \in v_r(P_i)$ *if and only if* r determines s.
2. $s \in v_r(\boldsymbol{N}P_j)$ *if and only if for some* $i \neq j$ $s \in v_r(P_i)$

Finally,

Definition 12 *A region* $\boldsymbol{S}$ *is* semantically indeterminate *relative to region* $\boldsymbol{R}$*, if, under the local interpretation induced by* $\boldsymbol{R}$, $\sim [P_i S \vee \boldsymbol{N}P_i S]$.

We can now prove the following theorem:

Theorem 1 *For regions R and S, if S is* relationally indeterminate *to R, then for all adequate* $\mathscr{L}_\psi$ *S is* semantically indeterminate *relative to R.*

Proof From definition 8, there is at least one state of s of S such that for all possible states of r of R, prob(s≡S|r≡R)≠1. Then, from definition 11, S is not in the extension of any predicate in $\mathbb{P}$ of the local interpretation induced by R, independently of the state of R. Nor is it in the extension of the predicate-negation of any of the predicates. Therefore, $\sim [P_i s \vee \mathbf{N}P_i s]$ for all P. Therefore, given that $\mathbb{P}$ is adequate and from definition 12 S is semantically indeterminate relative to R.

For a more intuitive presentation of the basic ideas here, consider a toy space-time theory in which the only values assigned to space-time are two scalar fields, A and B. Such a pair of scalar field assigns two real number values to each point of space-time. We can then define two sets of monadic predicates, A_i and B_i, as follows. Divide the possible values of each of the two fields into a collection of half-open sets, $\{[0,1), [1,2) \ldots [n,+\infty)\}$, where n is an arbitrary integer value. For simplicity assume that, for any particular application of the scheme, n is chosen so that the actual value of the relevant field is everywhere less than n. Associate a predicate with each element of the partition so that:

Definition 13 $A_n w$ *iff the value of the A-field is such that* $n \leq A(w) < (n+1)$

And, similarly for B. Clearly the set of predicates $\{A_n, B_n\}$ is adequate to this theory in the sense of Definition 10. Using the definitions from §§2 and 3 above, we can take the domain to be open sets on the space-time and the occupation function to be identity. Now, assume that we have a dynamics for A and B given by ordinary partial-differential equations with a well-formed initial value problem(IVP). Given the existence of a well-formed IVP, then when we specify the values of both fields on any single space-like hypersurface (Cauchy surface) there is a single *dynamically acceptable* extension to the remainder of space-time. But, for any region less than a complete Cauchy surface we have the same only for some spatio-temporal volume, less than the entire space-time, determined by the volume of the initial region. However, outside of that determinate region, A and B can take pretty much any values depending on the state of the remainder of space-time.

Define a state description for a region of space-time,

Definition 14 Ω_R*, is the set* $\{\sim B_1 R \ldots \sim B_{(i-1)} R,\ B_i R, \sim B_{(i+1)} R \ldots \sim B_n R,$
$\sim A_1 R \ldots \sim A_{(j-1)} R,\ A_j R, \sim A_{(j+1)} R \ldots \sim A_n R\}$
where $i \leq B(R) < (i+1)$ *and* $j \leq A(R) < (j+1)$.

Clearly a specification of the state of space-time, as above, determines a state description in this sense for every region of space-time. Just as straightforwardly, from the existence and uniqueness theorem for ordinary differential equations, the state description, Ω_C of a Cauchy surface possesses a single dynamically acceptable extension to a state-description, $\Omega_{\mathbb{W}}$ for the entire space-time. However, just as clearly there will be more than one dynamically acceptable extension of any region less than a complete Cauchy surface. There will, therefore, be at least two such state descriptions of at least some region compatible with the present state, thus inducing *semantic indeterminacy.*

But, despite various puzzles regarding the first-order representation of modern physical theories, I do not see any principled bar to defining such a language for any physical theory defined as above.[9]

Now consider Einstein-Minkowski space-time. Given a standard and natural reading of special relativity, where $\mathbb{C}(\mathrm{R})$ and $\mathbb{F}(\mathrm{R})$, the causally past and causally future regions of space-time are the past and future null-cones and their interiors respectively, then all regions of space-time that are not in the past relative to a given region are, in fact, relationally indeterminate(Harrington 2008).

5 Conclusion

In this paper we have explored a new model for the indeterminacy of the future. According to this model, objects to the future manifest attribute indefiniteness *via* predicate indeterminacy. Intuitively, this requires that there be at least some properties that such an object neither possess nor lacks. More formally, we represented this by introducing an aperator for predicate-term negation to an otherwise standard first-order language. A predicate in wuch a language is indeterminate in a model $\mathscr{M}$ just in case at least one element of the domaim of that model does belong neither to the extension of the predicate nor to the extension of its predicate negation.

This model has several interesting consequences. First, it provides at least as plausible a response to concerns about logical fatalism as do the the other non-fatalist accounts on offer. Second, it allows for an indeterminate future even against the backdrop of an eternalist ontology. Third, it provides a clear account of change, if we consider a temporally indexed series of models. Finally, and most importantly, it is a conception of indeterminacy straightforwardly compatible with ordinary physical theories of space-time.

Notes

[1]The term is taken from Adolf Grünbaum, although the definition and usage are mine.

[2]This possibility was first suggested to me a discussion of similar cases in (Prior and Fine 1977). The analogy with neo-Aristotelian term logic comes from Heinrich Wansing at Logica 2007. He also directed me to his useful essay, "Negation" (Wansing 2001)

[3]More formally, "It is not the case that, there is an event x, such that x occurs tomorrow and x is a sea-battle."

[4]Again we might reformulate this as an existential claim, rather than using names of events, as "There will be some event x such that x is a non sea-battle."

[5]Fortunately, John Earman has written just such an essay(Earman 2008). See Placek and Belnap (2010) for a response from the branching space-time camp.

[6]From here on out, the relationship of the state to some particular theory specifying the range of possible states will be left implicit.

[7]Technically, since the state space is likely to be continuous this would allow for for some measure zero set of states other than the one of interest to be compatible with S. I will follow the usual practice in physics and ignore this possibility. If one wished to significantly complicate the exposition at no substantive gain, one can replace all discussion of particular states with finite ranges of states.

[8]Technically a countable covering is required as long we retain the standard restriction on a countable number of terms in the corresponding language.

[9]Actually, I am eliding a whole class of problems in first-order representations of physical values, here. Including problems about values that vary along multiple dimensions; the nature of vector and tensor fields; and zero-value

physical quantities, just to name a few. For an interesting discussion of these issues, focused on the last problem, see Balashov (1999)

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